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Thermohydrodynamic Analysis for Laminar Lubricating Films

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1. INTRODUCTION

The purpose of this paper is to present a new method for incorporating thermal effects into the calculated performance of laminar lubricating films. There is enormous interest in the inclusion of such effects, as the recent reviews of Khonsari (1,2) attest. The reason for this interest is well founded, since the viscosity-temperature dependence of typical lubricants is such that the viscosity can vary many fold across and along a bearing film, with attendant effects on load capacity.

The up to date, extensive reviews in (1,2) remove the need here for a survey of prior literature, and reference will be made principally to those works used for comparative purposes. Suffice to say that earlier theoretical contributions on the subject of thermohydrodynamic lubrication divide themselves roughly into two categories. In the first category are those which embody a full transverse (cross-film) treatment of the energy equation using finite-difference or finite-element methods, and in the second category are those which incorporate rather drastic approximations to the transverse phenomena, usually representing the local film temperature distribution by a single value. Both approaches certainly possess merit. But the first approach obtains accuracy at the expense of computational speed, and the second obtains speed at the expense of accuracy.

We shall show here that if just <u>two</u> temperatures, chosen at "Lobatto points", are used to characterize the transverse temperature distribution in a laminar lubricating film, the effects of that distribution can be surprisingly well predicted. The calculations we have so-far performed have been directed solely towards demonstrating this fact, and only rudimentary numerical methods have been used in the plane of the film. Accordingly, no meaningful computation times can yet be reported. But we believe the technique will prove to be quite suitable for practical calculations.

In the present analysis, fluid properties are taken as constant and uniform, except for the viscosity. The flow is presumed to be laminar, with negligible inertia effects. The fluidity (reciprocal viscosity) is represented by a polynomial in terms of position across the film, with coefficients related to the local film temperature distribution. Use of this procedure permits a closed-form expression for the local lineal mass flux, albeit there is some difference between a fluidity profile which would everywhere correspond to the temperature profile and one which is thus approximated. The Lobatto-point temperatures, or mathematical equivalents, appear in two simultaneous partial differential equations obtained from the basic energy equation by a Galerkin procedure.

Implementation of the present approach has involved considerable tedious algebra, which, however, once done, causes no further embarrassment. The procedure should conveniently couple with cavitation algorithms, and preliminary testing indicates that no special handling is required to cope with moderate recirculation at film entry. Moreover, it can accommodate to some extent the temperature streaking from hot-oil carryover. We therefore expect to be able to exploit its use in a number of interesting directions.

2. NOMENCLATURE

C_p specific heat at constant pressure, J/kg-K

h film thickness, m

i Cartesian tensor index

j Cartesian tensor index

k thermal conductivity, $J/m^2-(K/m)$

$\dot{\bar{m}}$ lineal mass flux, kg/m-s

P_i Legendre polynomial, ith order ($P_0 = 1$; $P_1 = \zeta$; $P_2 = (3\zeta^2 - 1)/2$

p pressure, Pa

T temperature, K

t time, s

u x-wise velocity, m/s

v y-wise velocity, m/s

v_i i-th component of fluid velocity vector, m/s

w z-wise (cross-gap) velocity, m/s

w_i Lobatto weight function for i-th quadrature position, ζ_i

x lateral coordinate in direction of surface motion, m

y lateral coordinate transverse to surface motion, m

z coordinate perpendicular to gap midsurface, m

Greek:

- α fluidity functions (see Table I)
- β fluidity functions (see Table I)
- δ fluidity functions (see Table I)
- ϵ fluidity functions (see Table I)
- ζ dimensionless coordinate transverse to film, $2z/h$
- η fluid viscosity, N-s/m^2
- κ thermal diffusivity, m^2/s
- ξ fluidity functions (see Table I)
- ξ fluidity (reciprocal viscosity), m/N-s
- ρ fluid density, kg/m^3
- τ fluidity functions (see Table I)
- Φ dissipation function, J/m^3-s
- ϕ fluidity functions (see Table I)

3. BASIC EQUATIONS

In the absence of gravity, the momentum equation for a Newtonian fluid without dilational viscosity is:

$$\rho \frac{Dv_i}{Dt} = - \frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \eta \left\{ \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right\} \qquad [3.01]$$

where repeated subscripts imply summation. And the corresponding energy equation is:

$$\rho C_p \frac{DT}{Dt} + \frac{T}{\rho^2} \left(\frac{\partial \rho}{\partial T} \right)_p \frac{Dp}{Dt} = \frac{\partial}{\partial x_j} k \frac{\partial T}{\partial x_j} + \Phi \qquad [3.02]$$

where:

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + v_j \frac{\partial}{\partial x_j} \qquad [3.03]$$

is the time derivative following the fluid (Lagrangian derivative) and:

$$\Phi = \frac{\eta}{2} \left\{ \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right\}^2 \qquad [3.04]$$

is the dissipation function.

In lubricating films, lateral diffusion of momentum and heat is usually much less than transverse. Furthermore, inertia and pressure-energy effects are frequently negligible, and the transverse variation of pressure is small. Therefore, we adopt the following equations for laminar lubricating films.

$$0 = -\vec{\nabla} p + \frac{\partial}{\partial z} \eta \frac{\partial \vec{v}}{\partial z} \qquad [3.05]$$

and:

$$\rho C_p \frac{DT}{Dt} = \frac{\partial}{\partial z} k \frac{\partial T}{\partial z} + \eta \left(\frac{\partial \vec{v}}{\partial z} \right)^2 \qquad [3.06]$$

In these equations we shall treat the fluid viscosity as temperature dependent, and treat other fluid properties as constant.

To these equations must be added the mass continuity equation for an incompressible fluid. Thus:

$$\vec{\nabla} \cdot \vec{v} = 0 \qquad [3.07]$$

The coordinate system used with these equations is defined in Fig. 1. For convenience, a reference surface is taken midway between the film walls. A local coordinate system is substituted for a fixed Cartesian system, with the local x-y plane tangent to this reference surface. The film walls are rigid, but may be moving.

The Galerkin-style analysis used here involves the expansion of the temperature in a truncated series of Legendre polynomials. Satisfaction is required of as many moments of the energy equation as there are unknowns in this series. The ensuing partial differential equations for the Legendre components are then solved. In the present treatment, only two unknown components are used. And for these it is feasible to carry out explicit integration, as follows:

$$\frac{\partial}{\partial t} \int T \, dz + \frac{\partial}{\partial x} \int uT \, dz + \frac{\partial}{\partial y} \int vT \, dz = \kappa \left\{ \left(\frac{\partial T}{\partial z} \right)_2 - \left(\frac{\partial T}{\partial z} \right)_{-2} \right\} + \frac{1}{\rho C_p} \int \Phi \, dz \qquad [3.08]$$

$$\frac{\partial}{\partial t} \int zT \, dz + \frac{\partial}{\partial x} \int zuT \, dz + \frac{\partial}{\partial y} \int zvT \, dz - \int wT \, dz = \kappa \left\{ \frac{h}{2} \left[\left(\frac{\partial T}{\partial z} \right)_2 + \left(\frac{\partial T}{\partial z} \right)_{-2} \right] - T_2 - T_{-2} \right\} + \frac{1}{\rho C_p} \int z\Phi \, dz \qquad [3.09]$$

All of the above integrals are taken from the "bottom" to the "top" of the film. The subscripts $_2$ and $_{-2}$ are used to denote the upper and lower walls, respectively. The coordinate "z" is measured from the midplane. Continuity has been used to convert transverse velocity constructs to lateral constructs, wherever possible.

4. LOBATTO POINTS; DISTRIBUTION FORMULAS

An expression for the lineal mass flux can be developed directly from Eq. [3.05]. (See, for example, Dowson and Hudson (3)). But the convective terms in Eq. [3.06] involve integrals of the velocity-temperature product, and so require detailed knowledge of the respective distributions. Such information for the velocity is already available. For the temperature, we must develop our own expression.

Consider the integral $\int uT \, dz$ which relates to the cup-mean energy flow in the x-direction. Let the temperatures on the two walls be known, and the velocity be available where needed. If two sampling positions for the temperature -- and only two -- are to be allowed for estimating this integral, where should these positions be chosen? Without knowledge of end-point values,

it is best to choose the well-known Gaussian quadrature points. With knowledge of the end-point values, the optimum locations are the so-called "Lobatto points" (4), N interior points permitting exact numerical integration of a polynomial of degree (2N + 1). The case N = 1 yields Simpson's rule. Here we take N = 2, but it is evident that the procedure to be used is quite general, and that further refinement is possible with further analytical effort.

In terms of the variable $\zeta = 2z/h$, the Lobatto points providing exact numerical integration over the interval -1 to +1 of polynomials up to the fifth degree are:

Location, ζ_i	Weight, w_i	Subscript
1	1/6	2
$1/\sqrt{5}$	5/6	1
$-1/\sqrt{5}$	5/6	-1
1	1/6	-2

Then, for example, by Lobatto numerical quadrature:

$$\int_{-1}^{1} \zeta^4 \, d\zeta = 0.4 \qquad \text{(exact = 2/5)}$$

$$\int_{-1}^{1} \zeta^5 \, d\zeta = 0 \qquad \text{(exact = 0)}$$

$$\int_{-1}^{1} \zeta^6 \, d\zeta = 0.347 \qquad \text{(exact = 2/7)}$$

The temperature distribution which passes through the Lobatto-point values is most easily expressed in terms of an expansion in Legendre polynomials. Thus, if we write:

$$T(\zeta) = \sum_{k=0}^{3} \tilde{T}_k P_k(\zeta) \qquad [4.01]$$

then the Legendre coefficients are easily evaluated by integration.

$$\int_{-1}^{1} T(\zeta) P_k(\zeta) \, d\zeta = \frac{2}{2k+1} \tilde{T}_k \qquad [4.02]$$

Or:

$$\tilde{T}_k = \frac{2k+1}{2} \sum_{i=0}^{3} w_i T_i P_k(\zeta_i) \qquad [4.03]$$

The linear set of equations in [4.03] can be solved for the T_i, providing us with two modes of description of the temperature distribution in the lubricating film. The T_i give us detail, and the $\tilde{T}_i$ give us overall properties. In particular, $\tilde{T}_0$ is the space-mean temperature at the point (x,y) and $\tilde{T}_1$ is the first moment.

For N = 2:

$$\tilde{T}_0 = \frac{1}{12} (T_{-2} + 5T_{-1} + 5T_1 + T_2) \qquad [4.04]$$

$$\tilde{T}_1 = \left\{ T_2 - T_{-2} + \sqrt{5} \, (T_1 - T_{-1}) \right\} \Big/ 4 \qquad [4.05]$$

$$\tilde{T}_2 = \frac{5}{12} \left\{ T_2 + T_{-2} - (T_1 + T_{-1}) \right\} \qquad [4.06]$$

$$\tilde{T}_3 = \left\{ T_2 - T_{-2} - \sqrt{5} \, (T_1 - T_{-1}) \right\} \Big/ 4 \qquad [4.07]$$

In these expressions, the wall temperatures T_2 and T_{-2} are considered as known for purposes of the film calculation. It is then useful to note that:

$$\tilde{T}_2 = \frac{T_2 + T_{-2}}{2} - \tilde{T}_0 \qquad [4.08]$$

$$\tilde{T}_3 = \frac{T_2 - T_{-2}}{2} - \tilde{T}_1 \qquad [4.09]$$

An expression for the fluidity might be developed a number of ways. Here we collate the fluidity with the temperatures at the Lobatto points; that is, at the walls and at two interior points. The Legendre expansion for the fluidity is then developed in a manner completely analogous to that for the temperature. For example,

$$\tilde{\xi}_0 = (\xi_{-2} + 5\xi_{-1} + 5\xi_1 + \xi_2)/12 \qquad [4.10]$$

and the fluidity distribution is:

$$\xi = \sum_{k=0}^{3} \tilde{\xi}_k P_k(\zeta) \qquad [4.11]$$

5. VELOCITY EXPRESSIONS; MASS FLUX

A double integration of Eq. [3.05] (with $\xi = 1/\eta$) gives the tangential velocity vector. Thus:

$$\vec{v} = \vec{V}_{-2} + \vec{A} \int_{-1}^{\zeta} \xi \, d\zeta + \vec{B} \int_{-1}^{\zeta} \zeta\xi \, d\zeta \qquad [5.01]$$

$$\vec{A} = \frac{\vec{V}_2 - \vec{V}_{-2} - \vec{B} \int_{-1}^{1} \zeta\xi \, d\zeta}{\int_{-1}^{1} \xi \, d\zeta} \qquad [5.02]$$

and:

$$\vec{B} = \left(\frac{h}{2}\right)^2 \vec{\nabla}p \qquad [5.03]$$

In view of the Legendre expansion for ξ, [5.02] can be alternatively written as:

$$\vec{A} = \frac{\vec{V}_2 - \vec{V}_{-2} - \frac{2}{3} \tilde{\xi}_1 \vec{B}}{2\tilde{\xi}_0} \qquad [5.04]$$

Now to obtain the lineal mass flux, the velocity expression [5.01] is integrated again across the passage, with the result:

$$\frac{\dot{\vec{m}}}{\rho} = (\vec{V}_2 + \vec{V}_{-2}) \frac{h}{2} - \frac{2}{3} \tilde{\xi}_1 \vec{A} \frac{h}{2} - \frac{2}{3} \vec{B} (\tilde{\xi}_0 + \frac{2}{5} \tilde{\xi}_2) \qquad [5.05]$$

Simplification gives the following more recognizable expression:

$$\frac{\dot{\vec{m}}}{\rho} = (\vec{v}_2 + \vec{v}_{-2})\frac{h}{2} - (\vec{v}_2 - \vec{v}_{-2})\frac{h}{2}\left(\frac{1}{3}\frac{\tilde{\xi}_1}{\tilde{\xi}_o}\right) - \frac{h^3}{12}\vec{\nabla}p\left(\frac{\sum_{i=0}^{3}\xi_i}{4} - \frac{1}{3}\frac{\tilde{\xi}_1}{\tilde{\zeta}_o}^2\right) \qquad [5.06]$$

Here the fluidity parameters $\tilde{\xi}_i$ are the vehicle for the temperature-flow interaction. The parameter $\tilde{\xi}_1$ represents asymmetry of the fluidity distribution. It is interesting to note that for symmetric temperature (and fluidity) distributions, the sole effect of temperature on the mass flow is through the arithmetic average of the fluidities at the Lobatto points. This result is a special case of the following formula applicable for any symmetric cross-film temperature distribution:

$$\frac{\dot{\vec{m}}}{\rho} = \left(\vec{v}_2 + \vec{v}_{-2}\right)\frac{h}{2} - \frac{h^3}{12}\vec{\nabla}p\left|\frac{3}{2}\int_{-1}^{1}\zeta^2\xi\,d\zeta\right| \qquad [5.07]$$

For such cases, Eq. [5.07] justifies the effective viscosity concept, and shows the relative importance of the temperatures near the walls.

Mass continuity applied to the mass flux expression [5.06] leads to a generalized Reynolds equation, the divergence of the mass flux involving spatial derivatives of pressure. Thus:

$$\frac{\partial h}{\partial t} + \vec{\nabla}\cdot\frac{\dot{\vec{m}}}{\rho} = 0 \qquad [5.08]$$

The first moment of the energy equation involves the cross velocity, w. With a little care, this velocity can be found via mass continuity. We have:

$$\frac{\partial w}{\partial z} = -\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) \equiv (\vec{\nabla}\cdot\vec{v})_z \qquad [5.09]$$

Transforming coordinates from (x,y,z) to (x,y,ζ) we find:

$$(\vec{\nabla}\cdot\vec{v})_z = (\vec{\nabla}\cdot\vec{v})_\zeta - \frac{\zeta}{h}\vec{\nabla}h\cdot\frac{\partial\vec{v}}{\partial\zeta} \qquad [5.10]$$

(Here the velocity vector is understood to consist only of (u,v).) Substituting this result into [5.09] and integrating, we get:

$$w = -\int_{-h/2}^{z}\left|\vec{\nabla}_\zeta\cdot\vec{v} - \frac{\zeta}{h}\vec{\nabla}h\cdot\frac{\partial\vec{v}}{\partial\zeta}\right|dz + w_{-2} \qquad [5.11]$$

Finally, integrating by parts, we obtain:

$$w = \left(w_{-2} + \vec{v}_{-2}\cdot\vec{\nabla}\frac{h}{2}\right) + \vec{\nabla}\frac{h}{2}\cdot\zeta\vec{v} - \vec{\nabla}_\zeta\cdot\frac{h}{2}\int_{-1}^{\zeta}\vec{v}\,d\zeta \qquad [5.12]$$

The expressions for the tangential and cross velocities will be essential for evaluating the contributions to convective heat transfer.

6. THE TEMPERATURE EQUATIONS

With the aid of Legendre series for the temperature and fluidity, we are now in a position to evaluate the integrals appearing in the zeroth and first moment of the energy equation; namely, Eqs. [3.03] and [3.09]. Implementation is straightforward, but tedious. All requisite coefficients are given in Table I.

Equation [3.08] becomes:

$$\frac{\partial}{\partial t}(h\tilde{T}_o) + \vec{\nabla}\cdot\frac{\dot{\vec{m}}}{\rho}\tilde{T}_o = \frac{12\kappa}{h}\left(\frac{T_2 + T_{-2}}{2} - \tilde{T}_o\right) + \frac{\int\Phi\,d\zeta}{\rho C_p} - C_o \qquad [6.01]$$

where "C_o" stands for the collection of terms:

$$\vec{\nabla}\cdot\left(\vec{A}\sum_{k=0}^{3}\alpha_k\tilde{T}_k + \vec{B}\sum_{k=0}^{3}\phi_k\tilde{T}_k\right) \qquad [6.02]$$

The temperature $\tilde{T}_o$ appearing here is the ζ-space mean temperature, and not the mixed-cup temperature.

The integral of the dissipation function is:

$$\int\Phi\,dz = \frac{2}{h}\left\{2\tilde{\xi}_o A^2 + \frac{2}{3}\tilde{\xi}_1(2\vec{A}\cdot\vec{B}) + \frac{2}{3}(\tilde{\xi}_o + \frac{2}{5}\tilde{\xi}_2)B^2\right\} \qquad [6.03]$$

Equation [3.09] becomes:

$$\frac{\partial}{\partial t}(h^2\tilde{T}_1) = 60\kappa\left|\frac{T_2 - T_{-2}}{2} - \tilde{T}_1\right| + \frac{6\left(\frac{h}{2}\right)^2}{\rho C_p}\int_{-1}^{1}\zeta\Phi\,d\zeta - 6\,C_1 \qquad [6.04]$$

where "C_1" stands for the terms:

$$C_1 = -h\tilde{T}_o w_{-2} + \vec{\nabla}\cdot\frac{h^2\tilde{T}_1}{6}\vec{v}_{-2} + \frac{h}{2}\vec{\nabla}\cdot\frac{h}{2}\left(\vec{A}\sum_{k=0}^{3}\gamma_k\tilde{T}_k + \vec{B}\sum_{k=0}^{3}\delta_k\tilde{T}_k\right) + \frac{h}{2}\sum_{k=0}^{3}\tilde{T}_k\vec{\nabla}_\zeta\cdot\frac{h}{2}(\vec{A}\alpha_k + \vec{B}\beta_k) \qquad [6.05]$$

The moment of the dissipation function is:

$$\int_{-1}^{1}\zeta\Phi\,d\zeta = \frac{2}{3}\tilde{\xi}_1 A^2 + \frac{2}{3}(\tilde{\xi}_o + \frac{2}{5}\tilde{\xi}_2)(2\vec{A}\cdot\vec{B}) + \frac{2}{5}(\tilde{\xi}_1 + \frac{2}{7}\tilde{\xi}_3)B^2 \qquad [6.06]$$

Equations [6.01] and [6.04] are two simultaneous partial differential equations in the two variables, $\bar{T}_0$ and $\bar{T}_1$. Where they appear, $\bar{T}_2$ and $\bar{T}_3$ can be eliminated via Eqs. [4.08] and [4.09] in favor of these dependent variables. Coupled with the generalized Reynolds Eq. [5.07], they provide our approximate thermohydrodynamic treatment for laminar films.

As mentioned previously, our numerical techniques for dealing with these differential equations have so far been extremely simple. The steady-state solutions to be presented in the next section were found by solving the foregoing equations in their transient form, and no study has been made of the possibilities for economy in the fluidity evaluations. Prior to this work, enough success has been obtained by others with analyses which neglect entirely any cross-film viscosity variations so that it seems unlikely that it will prove necessary to update the terms in Table I at every step of a solution.

7. RESULTS FOR THE INFINITELY-WIDE SLIDER BEARING

In 1963, Dowson and Hudson (3) performed some finite-difference calculations on the infinitely-wide flat slider bearing, including variable-property effects. They employed 100 increments along the length of the bearing, and a minimum of 20 increments transversely. These investigators were concerned with the relative effects of density and viscosity variations upon load capacity, with the effects of solid-fluid thermal interaction, and with the possibilities for load support from parallel surfaces. Their findings serve as basis for our assumption of a constant-density fluid, and two special cases of their exploratory calculations will be used here for direct numerical comparisons.

The fluid properties used by Dowson and Hudson were as follows (SI units):

Density: 1.7577×10^6 kg/m^3
Thermal diffusivity: 7.306×10^{-8} m^2/s
Viscosity: $\eta = 0.13885 * \exp(-0.045(T-311.11))$ PaS
Lubricant entrance temperature: 311.11 K

Wall temperatures: Uniformly at 311.11 K
Runner velocity: 31.946 m/s
Bearing length: 0.18288 m
Minimum gap: 0.00009144 m

In the first case for comparison, the film thickness ratio is 2/1. Figures 2 and 3 show the velocity and temperature profiles obtained by us at the entrance, at 0.65 of the length, and at the exit. Figure 4 shows the corresponding pressure profile along the length of the bearing. Included in this last figure is the profile that would result if the entrance value of viscosity persisted throughout the film. The circles were read from the Dowson-Hudson curves, and the agreement is almost within reading accuracy. Figure 5 compares predicted temperature contours.

In the second case, the bearing surfaces are parallel. The possibility of load support through a "viscosity wedge" is being explored. Figure 6 compares pressure distributions. Again, excellent agreement is achieved. We note parenthetically that Dowson and Hudson demonstrated that, with more realistic wall boundary conditions for the temperature, the above-shown load support vanishes.

Finally, Figs. 7 to 9 show the velocity, temperature and pressure profiles for a flat slider with 4/1 film-thickness ratio. These calculations were performed to test the sensitivity of the analysis to flow reversal at the entrance. As mentioned earlier, such reversal can cause problems for point-by-point prediction methods. We have yet to make any comparisons with other investigations.

8. CONCLUSIONS

A Galerkin-type analysis has been made of temperature effects in laminar lubricating films. The procedure capitalizes on the suitability of so-called "Lobatto points" for sampling of the temperature distribution. Preliminary indications are that the use of just two such sampling points enables satisfactory prediction of bearing performance even in the presence of substantial viscosity variation.

The procedure described herein yields two partial differential equations, one for the local space-mean temperature, and one for the first transverse moment of the temperature distribution. These temperature equations are coupled to a generalized Reynolds equation. Results have been presented for the steady-state, infinitely-wide flat slider bearing, and comparisons with the earlier, detailed work of Dowson and Hudson are very encouraging. The procedure is quite general, and our intent is now to refine the numerical techniques, and to carry out calculations for more realistic configurations and boundary conditions.

9. ACKNOWLEDGMENTS

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TABLE I. - FLUIDITY FUNCTIONS

$$\xi_1 = \xi(T_1)$$
$$\xi_2 = \xi(T_2)$$
$$\xi_{-1} = \xi(T_{-1})$$
$$\xi_{-2} = \xi(T_{-2})$$

$$\tilde{\xi}_0 = (\xi_2 + 5\xi_1 + 5\xi_{-1} + \xi_{-2})/12$$
$$\tilde{\xi}_1 = (\xi_2 - \xi_{-2} + \sqrt{5}[\xi_1 - \xi_{-1}])/4$$
$$\tilde{\xi}_2 = (\xi_2 + \xi_{-2} - \xi_1 - \xi_{-1})5/12$$
$$\tilde{\xi}_3 = (\xi_2 - \xi_{-2} - \sqrt{5}[\xi_1 - \xi_{-1}])/4$$

$$\alpha_0 = 2(2\tilde{\xi}_0 - \tilde{\xi}_1 + \tilde{\xi}_2/5)/3$$
$$\alpha_1 = 2(\tilde{\xi}_0 - 2\tilde{\xi}_1/5 + \tilde{\xi}_3/35)/3$$
$$\alpha_2 = 2(\tilde{\xi}_0 - 2\tilde{\xi}_2/7)/15$$
$$\alpha_3 = 2(\tilde{\xi}_1 - 2\tilde{\xi}_3/3)/105$$

$$\gamma_0 = 2(\tilde{\xi}_0 - \tilde{\xi}_2/5)/3$$
$$\gamma_1 = 2(5\tilde{\xi}_0 - \tilde{\xi}_1 - 2\tilde{\xi}_3/7)/15$$
$$\gamma_2 = 2(2\tilde{\xi}_0 - \tilde{\xi}_2/7)/15$$
$$\gamma_3 = 2(\tilde{\xi}_1 - \tilde{\xi}_3/9)/35$$

$$\beta_0 = 2(-5\tilde{\xi}_0 + 4\tilde{\xi}_1 - 2\tilde{\xi}_2 + 3\tilde{\xi}_3/7)/15$$
$$\beta_1 = 2(-2\tilde{\xi}_0/5 + \tilde{\xi}_1/3 - \tilde{\xi}_2/7)/3$$
$$\beta_2 = 2(\tilde{\xi}_1 - 2\tilde{\xi}_3/3)/105$$
$$\beta_3 = 2\tilde{\xi}_0/105$$

$$\delta_0 = 2(\tilde{\xi}_1/3 - \tilde{\xi}_3/7)/5$$
$$\delta_1 = 2(-\tilde{\xi}_0/5 + \tilde{\xi}_1/3 - 4\tilde{\xi}_2/35)/3$$
$$\delta_2 = 2(4\tilde{\xi}_1 - \tilde{\xi}_3)/105$$
$$\delta_3 = 2(\tilde{\xi}_0 + \tilde{\xi}_2/3)/35$$

$$\phi_0 = 0$$
$$\phi_1 = 2(\tilde{\xi}_1/3 - \tilde{\xi}_3/7)/5$$
$$\phi_2 = 2(\tilde{\xi}_0 + \tilde{\xi}_2/7)/15$$
$$\phi_3 = 2(2\tilde{\xi}_1 + \tilde{\xi}_3/3)/105$$

$$\epsilon_0 = 0$$
$$\epsilon_1 = 2(\tilde{\xi}_0 - \tilde{\xi}_2/5)/3$$
$$\epsilon_2 = 2(\tilde{\xi}_1/3 - \tilde{\xi}_3/7)/5$$
$$\epsilon_3 = 2\tilde{\xi}_2/35$$

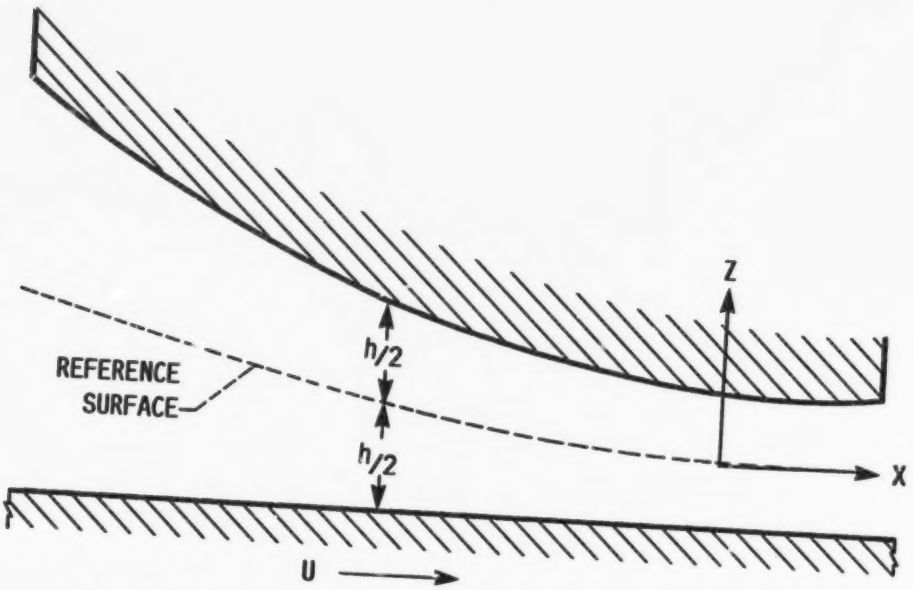

FIGURE 1.- COORDINATE DEFINITIONS FOR SLIDER BEARING.

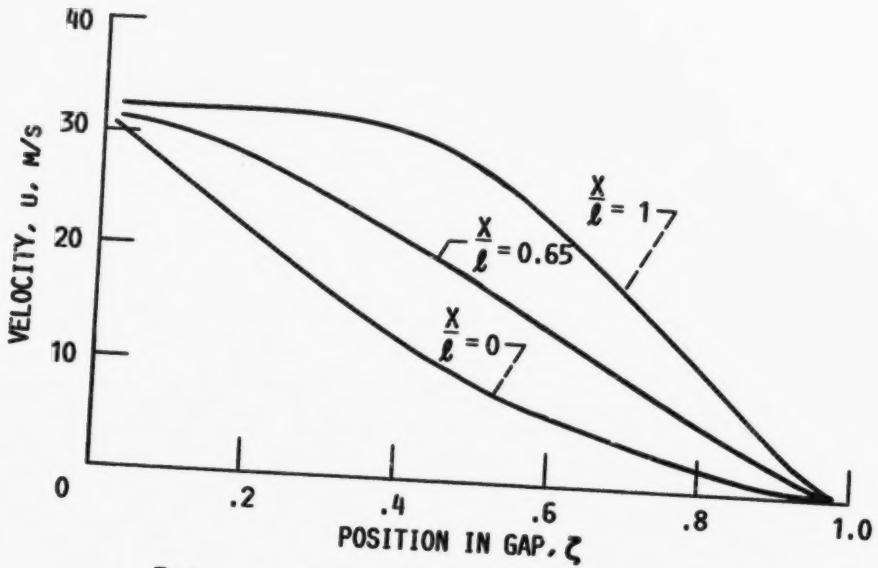

FIGURE 2.- X-VELOCITY VERSUS POSITION IN GAP.

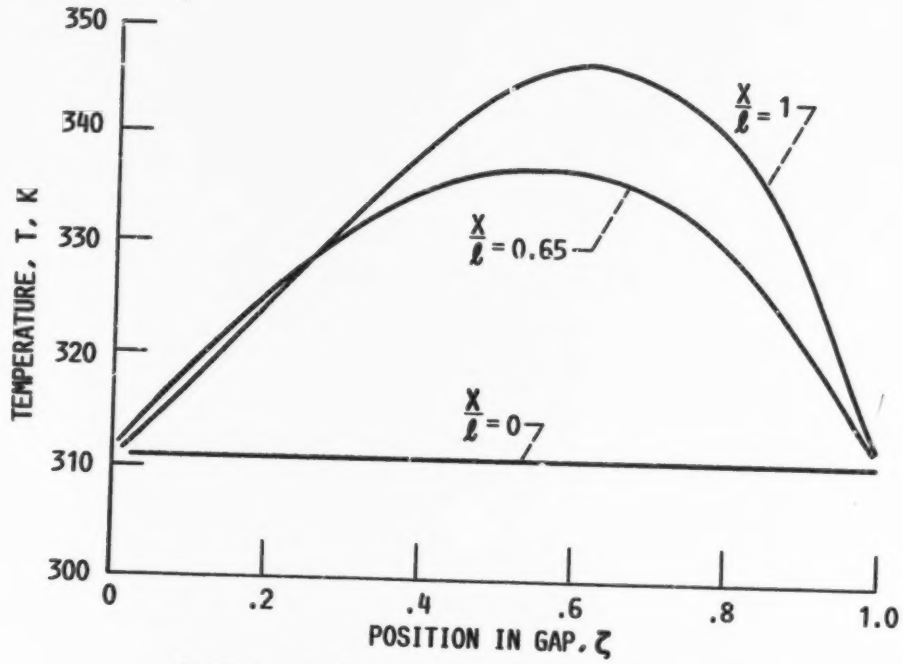

FIGURE 3.- TEMPERATURE VERSUS POSITION IN GAP.

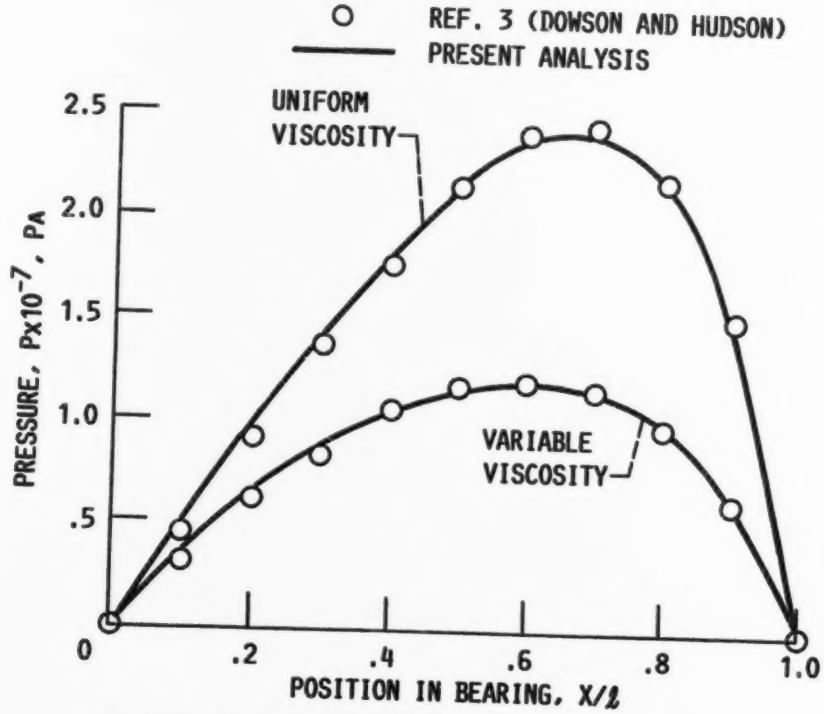

FIGURE 4.- PRESSURE VERSUS POSITION IN BEARING.

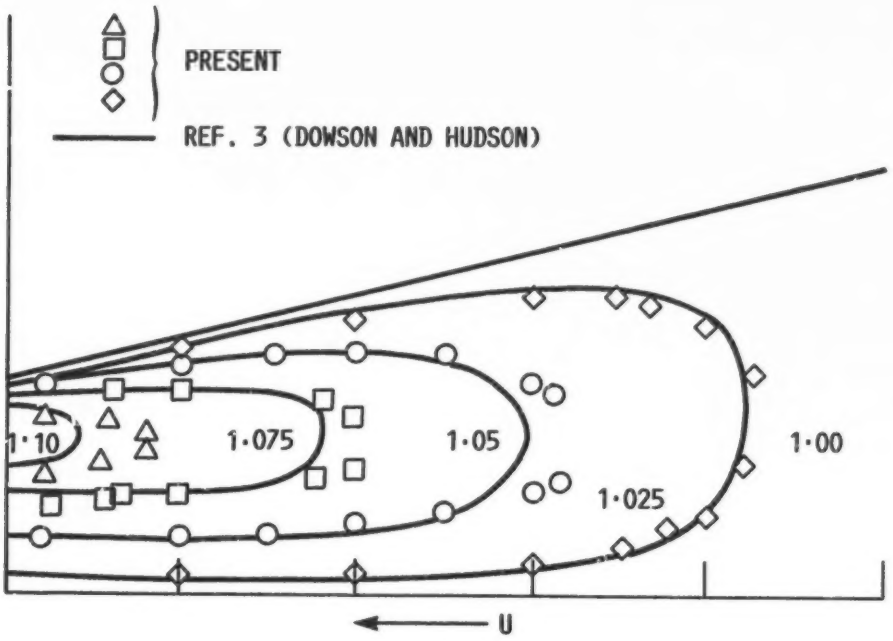

FIGURE 5.- TEMPERATURE CONTOURS, T/T_W.

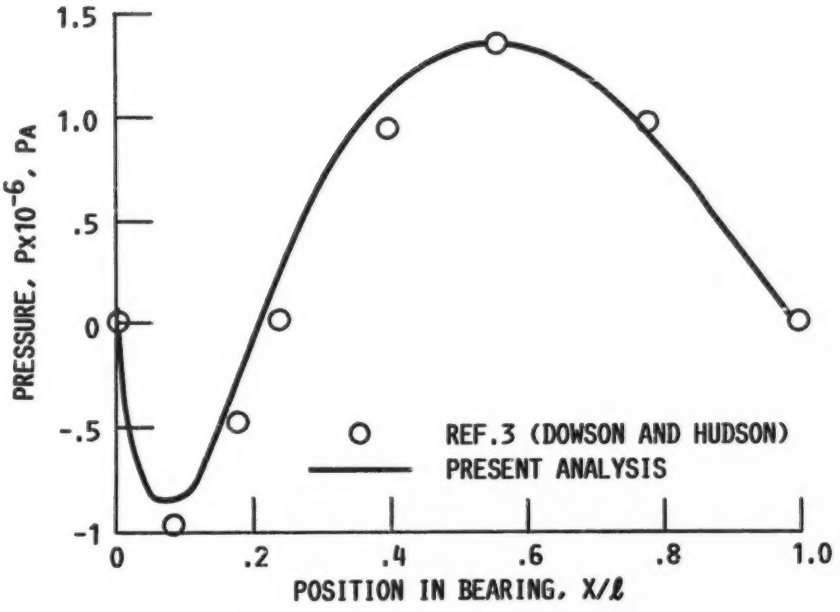

FIGURE 6.- PRESSURE VERSUS POSITION IN BEARING.

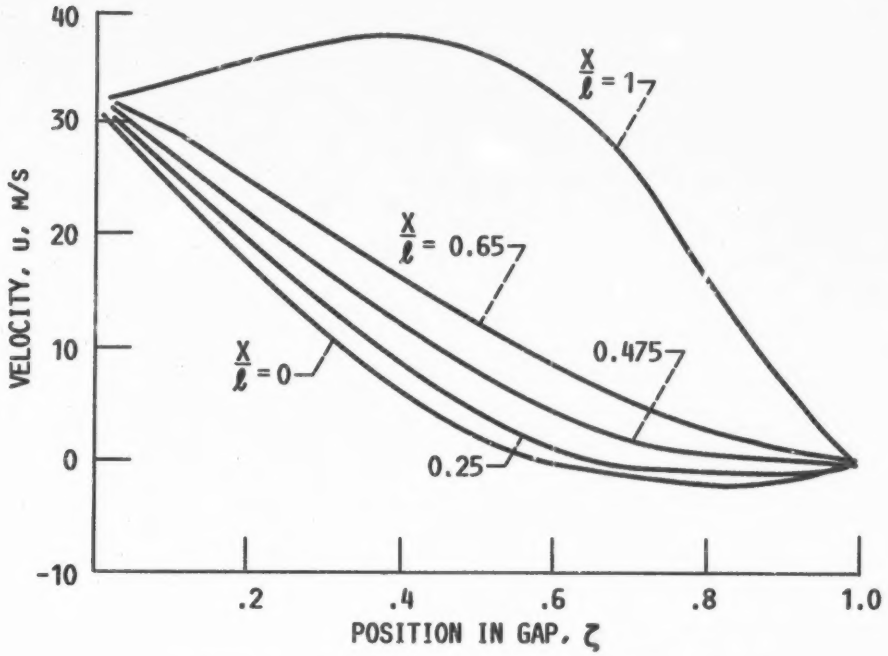

FIGURE 7.- X-VELOCITY VERSUS POSITION IN GAP.

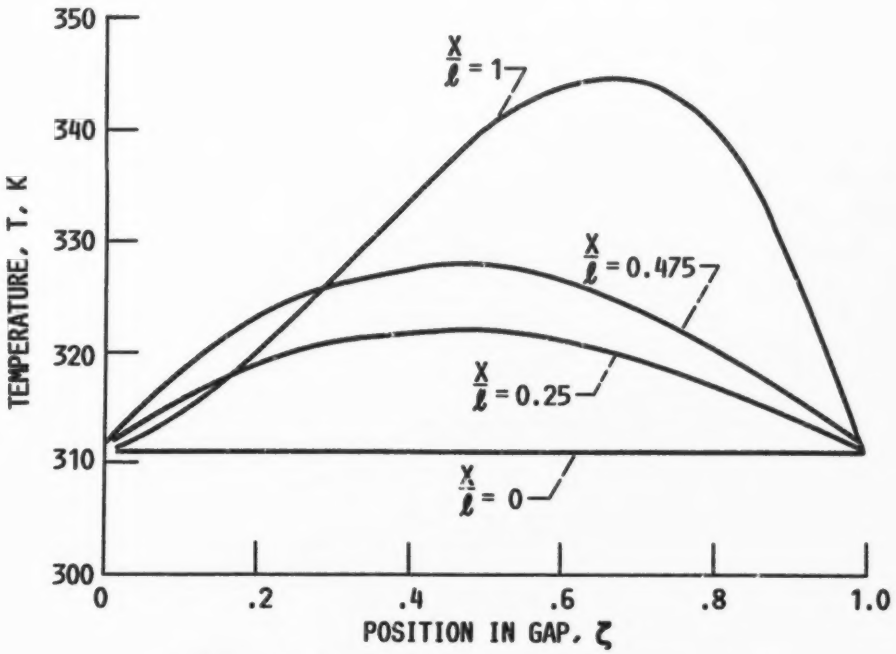

FIGURE 8.- TEMPERATURE VERSUS POSITION IN GAP.

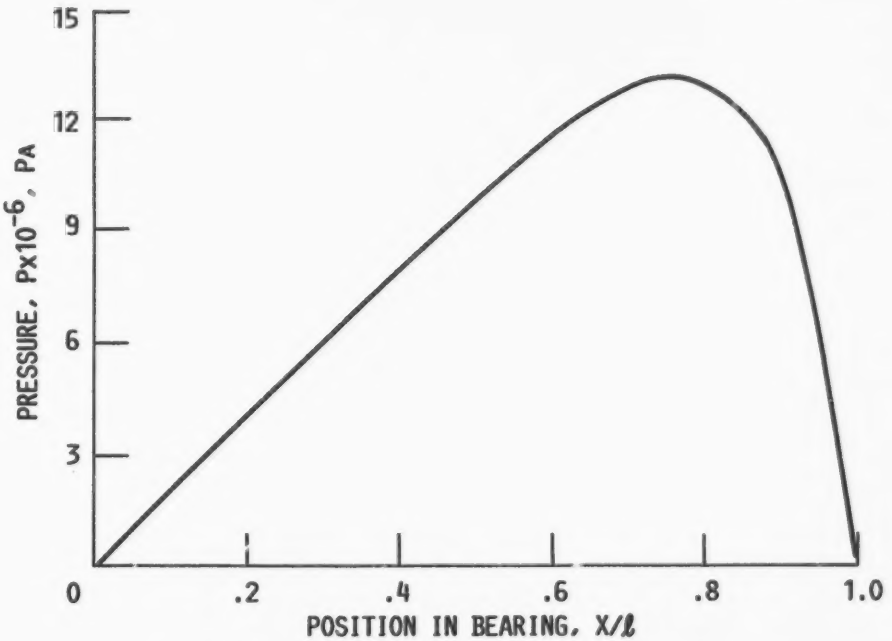

FIGURE 9.- PRESSURE VERSUS POSITION IN BEARING.

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15. Supplementary Notes

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16. Abstract

A Galerkin-type analysis to include thermal effects in laminar lubricating films was performed. The lubricant properties were assumed constant except for a temperature-dependent Newtonian viscosity. The cross-film temperature profile is established by collocation at the film boundaries and two interior Lobatto points. The interior temperatures are determined by requiring the zeroth and first moment of the energy equation be satisfied across the film. The fluidity is forced to conform to a third-degree polynomial appropriate to the Lobatto-point temperatures. Preliminary indications are that the use of just two such sampling points enables satisfactory prediction of bearing performance even in the presence of substantial viscosity variation.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Thermal film; Thermohydrodynamic; Lubricants; Thermal effects; Laminar films; Slider bearing; Lobatto points	Unclassified - unlimited STAR Category 34

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